Optimal data fusion of radar and passive microwave measurements in the estimation of vertical rain profiles

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It is easy to understand heuristically why single- frequency spaceborne radar measurements, by themselves, are not sufficient to solve the inverse problem of retrieving an exact underlying rain-rate. profile. On one hand, we have the unknown quantities consisting of the continuous rain-rate function (of range), along with several additional continuous or discrete rain parameters, while on the other hand the data consist of a single continuous function: the measured radar reflectivity profile. Too many unknowns with too few equations. Several schemes have been proposed to create additional equations out of the brightness temperatures measured by a passive microwave radiometer, thus generically guaranteeing that the inversion problem can be solved if one has as many radiometer channels as there are additional unknown rain parameters. Typical schemes use one passive measurement to determine one additional discrete rain variable. The results reported typically indicate that the solutions obtained using different radiometer channels do not coincide.

Discrepancies such as these are not unexpected: the models used in practice to describe the rain and its electromagnetic properties are but rough approximations of the complex physics taking place - and the measurements themselves are noisy. Therefore, if one is looking for an exact solution to the inverse problem, then starting with the same number of unknowns as one has equations, one may well find a unique solution, but it may or may not be close to the actual values that gave list to the observations, depending on the sources of error and their specific effect. If one started With fewer (or more) unknowns than equations, one would generically end up with discrepancies between the theoretically predicted values and the actual observations, with no well-defined procedure to reconcile the mismatches unless one accounts for the sources of error.

This is precisely the kind of problem which a Bayesian approach can help solve optimally. If we write $\vec{R}$ to represent all the unknown quantities describing the rain, and $\vec{T}_b$ for all the passive microwave observations, $\vec{T}_{radar}$ for all the radar reflectivity observations of the rain column, the Bayesian approach would compute the probability density function for $\vec{R}$ given all the observations available ($\vec{T}_b$ and $\vec{T}_{radar}$). The conditional p.d.f. for $\vec{R}$, conditioned on $\vec{T}_b$ and $\vec{T}_{radar}$, satisfies

$$\mathcal{P}(\vec{R}\,|\,\vec{T}_b,\vec{T}_{radar}) = \mathcal{P}(\vec{T}_b\,|\,\vec{R},\vec{T}_{radar})\cdot\frac{\mathcal{P}(\vec{R}\,|\,\vec{T}_{radar})}{\mathcal{P}(\vec{T}_b\,|\,\vec{T}_{radar})} = \mathcal{P}(\vec{T}_b\,|\,\vec{R},\vec{T}_{radar})\cdot\mathcal{P}(\vec{R}\,|\,\vec{T}_{radar}),$$

up to a normalization constant. This equation naturally suggests a two-step backward-forward procedure to compute the conditional density for $\vec{R}$. Indeed, we have developed an algorithm to compute the two factors in the right-hand-side, and hence the full conditional probability density function $\mathcal{P}$. Along with nonlinear filtering techniques, we use a numerically stable Hitschfeld-Bordan (inversion) algorithm to compute the second term, and a (forward) Eddington approximation to compute the first. The conditional means of $\mathcal{P}$ then give the estimates of the rain variables, and the conditional variances give the mean-squared uncertainties. We also compare the uncertainties using this approach with the uncertainties in the radar-only estimates.